

On the Quark Mass Dependence of Two Nucleon Observables

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Abstract

We study the implications of lattice QCD determinations of the S-wave nucleon-nucleon scattering lengths at unphysical light quark masses. It is found that with the help of nuclear effective field theory (NEFT), not only the quark mass dependence of the effective range parameters, but also the leading quark mass dependence of all the low energy deuteron matrix elements can be obtained. The quark mass dependence of deuteron charge radius, magnetic moment, polarizability and the deuteron photodisintegration cross section are shown based on the NPLQCD lattice calculation of the scattering lengths at 354 MeV pion mass and the NEFT power counting scheme of Beane, Kaplan and Vuorinen. Further improvement can be obtained by performing the lattice calculation at smaller quark masses. Our result can be used to constrain the time variation of isoscalar combination of u and d quark mass m_q , to help the anthropic principle study to find the m_q range which allows the existence of life, and to provide a weak test of the multiverse conjecture.

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I. INTRODUCTION

A very interesting aspect of lattice QCD (LQCD) calculations is that one can study the quark mass dependence of physical observables which are otherwise hard to measure with experiments. This information could be used to constrain the time variation [1–3] of quark masses in the evolution of the universe [4–7]. It could also shed light on how finely tuned the quark masses should be [8–12] such that light nuclei can be synthesized through the usual pathway of Big Bang Nucleosynthesis (BBN) [13, 14] and make the familiar carbon based life forms possible.

Much has been learned from the u and d quark mass (we will work in the isosymmetric limit $m_u = m_d = m_q$) dependence of the meson and single baryon observables [15, 16] through lattice QCD (LQCD) [17], chiral perturbation theory (ChPT) [18–21] and experimental data. In principle, lattice QCD can map out all the m_q dependence for these observables. However, most of the calculations are done with m_q 's larger than their physical values, because it requires more computing resources to work with smaller m_q . Fortunately, ChPT, which is an effective field theory (EFT) of QCD, can be used to describe the m_q dependence once the unknown parameters in the theory are fixed by either experiments or lattice data.

In the multi-baryon sector, much progress has been made in LQCD in two nucleon [22–26], nucleon-hyperon [27], triton [28, 29] and α -particle [29] systems (see [30] for a brief review). However, for two nucleon systems, so far only the S-wave scattering lengths have been computed with 354 MeV or heavier pion mass m_π . (Note that the physical pion mass $m_\pi^{phys} \simeq 138$ MeV, and there is a one-to-one correspondence between m_π and m_q , e.g. $m_\pi \propto m_q^{1/2}$ as $m_q \rightarrow 0$. So the m_π and m_q dependence can be converted to each other.) Even so, as will be demonstrated in this work, this information is enough to determine the leading m_π dependence of all the low energy matrix elements involving deuterons.

We will focus on processes with the typical momentum $p \ll m_\pi$, such that the pions can be taken as heavy particles and integrated out of the theory. This theory is known as pionless theory [31–34]. The information of the pion dynamics in the pionful theory is now encoded in the m_π dependent couplings of the pionless theory. It is found that, all the leading m_π dependence in deuteron matrix elements in the pionless theory can be computed using the pionful theory together with the m_π dependence of the S-wave

scattering lengths obtained from LQCD. Thus, once they are fixed at m_π^{phys} , their values at other pion masses are also known.

Of course, one can still work with the pionful theory. The matching is a convenient but not necessary step to take. One advantage of working with the theory without pions is that once the m_π dependence of the couplings are worked out, one can just perform the calculation in the pionless theory instead of the more complicated pionful theory. As an explicit example, we match the pionful theory based on Beane, Kaplan and Vuorinen's (BKV) [35] power counting scheme to a pionless theory. This allows the matching been done analytically. However, the method can be applied to other power schemes as well.

II. POWER COUNTING SCHEMES IN NUCLEAR EFFECTIVE FIELD THEORY

Currently, there are several power counting schemes for the nuclear effective theory used for multinucleon systems. Power counting means counting the power of the small expansion parameter of a Feynman diagram, such that one can organize the computation in a series expansion of this parameter. In nuclear EFT, the small expansion parameter Q is m_π/Λ and p/Λ , where Λ is the cut-off scale. Here we briefly review some popular power counting schemes.

In Weinberg's scheme [36–38], power counting is done to the potential of the Lippmann-Schwinger equation, not the diagram. The leading-order (LO) potential involves the one pion exchange (OPE) potential and the delta function potential from contact interactions. Subtracting the infinities in the LO diagram requires higher order operators with high power of quark mass insertions. Thus, the result has cut-off dependence that cannot be removed [39–41]. A similar situation happens to higher partial waves as well [41]. However, within a reasonable range of cut-off, the scheme works well numerically with impressive fits to nucleon-nucleon (NN) scattering phase shift data at the fourth order [42–58].

The alternative KSW scheme [59, 60] counts the diagrams near the non-trivial UV fixed point of the four-nucleon operators such that the cut-off dependence is removed and diagrams of the same order are of equal size. The LO S-wave diagrams only contain non-derivative four-nucleon contact interactions, while the next-to-leading-order (NLO) contains OPE diagrams and diagrams with higher order four-nucleon operators. However,

numerically, the convergence is not good in the 3S_1 channel due to the singular nature of the tensor pion exchange potential at short distance $\widehat{\mathbf{r}}_i\widehat{\mathbf{r}}_j/r^3$, where \mathbf{r} is the distance between two nucleons [61, 62]. This suggests that the tensor pion exchange might not be perturbative.

In view of this problem, the tensor pion exchange is resummed at the LO in the BBSvK scheme [40] where the 1S_0 channel follows the KSW power counting while the 3S_1 channel follows the Weinberg's power counting. It was shown that the cut-off can be removed in this scheme.

The BKV scheme [35] seeks to fix the same problem by introducing a Pauli-Villars (PV) field in the 3S_1 channel to remove the short distance part of the singular tensor potential. The resulting 3S_1 phase shift is convergent. The price to pay is that the PV mass λ is counted as the same order as m_π , but numerically it is close to the cut-off scale. However, its analytic result is very convenient to perform the matching to a pionless theory. Thus, we will adopt the BKV scheme in this work.

III. THE QUARK MASS DEPENDENCE OF EFFECTIVE RANGE PARAMETERS

The S-wave nucleon-nucleon (NN) scattering amplitude is

$$\mathcal{A} = \frac{4\pi}{M} \frac{1}{p \cot \delta - ip}, \quad (1)$$

where $M = 938.92$ MeV is the nucleon mass, p is the magnitude of the nucleon three-momentum in the center-of-mass (CM) frame and δ is the S-wave phase shift. If the interaction (potential) is localized, then δ has the expansion [63, 64]

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2}r_0p^2 + \dots, \quad (2)$$

where the effective range parameters (ERP's) a and r_0 are the scattering length and the effective range, respectively. The shape parameter and higher order terms are not shown.

In the BKV scheme, the amplitude can be expanded in powers of the small expansion parameter Q

$$\mathcal{A} = \mathcal{A}_{-1} + \mathcal{A}_0 + \mathcal{A}_1 + \dots, \quad (3)$$

where \mathcal{A}_n is of order Q^n in the expansion. Hence

$$p \cot \delta = ip + \frac{4\pi}{M\mathcal{A}_{-1}} - \frac{4\pi\mathcal{A}_0}{M\mathcal{A}_{-1}^2} + \dots \quad (4)$$

A. Working in the BKV scheme as an explicit example

The BKV scheme is the same as the KSW scheme in the 1S_0 channel but different in the 3S_1 channel. The leading order (LO) amplitude of channel i arises from the diagrams in Fig. 5 of Ref. [59]

$$\mathcal{A}_{-1}^{(i)} = \frac{-C_0^{(i)}}{\left[1 + \frac{C_0^{(i)}M}{4\pi}(\mu + ip)\right]} \quad , \quad (5)$$

where C_0 is the LO four-nucleon non-derivative coupling which is independent of m_q . The next-to-leading-order (NLO) amplitude arises from the diagrams in Fig. 6 of Ref. [59] plus the associated diagrams with the Pauli-Villars fields

$$\begin{aligned} \mathcal{A}_0^{(^1S_0)} &= \mathcal{A}_{0,a}^{(^1S_0)} + \mathcal{A}_{0,b}^{(^1S_0)}(m_\pi) \\ \mathcal{A}_0^{(^3S_1)} &= \mathcal{A}_{0,a}^{(^3S_1)} + \mathcal{A}_{0,b}^{(^3S_1)}(m_\pi) - \epsilon \mathcal{A}_{0,b}^{(^3S_1)}(\lambda) \end{aligned} \quad (6)$$

where ϵ is introduced to keep track of the difference between the BKV and the KSW power counting. $\epsilon = 1$ gives the BKV result while $\epsilon = 0$ gives the KSW result.

$$\begin{aligned} \mathcal{A}_{0,a}^{(i)} &= \frac{-\left(C_2^{(i)}p^2 + C_{0,0}^{(i)}\right)}{\left[1 + \frac{C_0^{(i)}M}{4\pi}(\mu + ip)\right]^2} \\ \mathcal{A}_{0,b}^{(i)}(m_\pi) &= \frac{-D_2^{(i)}m_\pi^2}{\left[1 + \frac{C_0^{(i)}M}{4\pi}(\mu + ip)\right]^2} \\ &+ \left(\frac{g_A^2}{2f^2}\right) \left(-1 + \frac{m_\pi^2}{4p^2} \ln\left(1 + \frac{4p^2}{m_\pi^2}\right)\right) \\ &+ \frac{g_A^2}{f^2} \left(\frac{m_\pi M \mathcal{A}_{-1}}{4\pi}\right) \left\{ -\frac{(\mu + ip)}{m_\pi} + \frac{m_\pi}{2p} \left[\tan^{-1}\left(\frac{2p}{m_\pi}\right) + \frac{i}{2} \ln\left(1 + \frac{4p^2}{m_\pi^2}\right) \right] \right\} \\ &+ \frac{g_A^2}{2f^2} \left(\frac{m_\pi M \mathcal{A}_{-1}}{4\pi}\right)^2 \left\{ 1 - \left(\frac{\mu + ip}{m_\pi}\right)^2 + i \tan^{-1}\left(\frac{2p}{m_\pi}\right) - \frac{1}{2} \ln\left(\frac{m_\pi^2 + 4p^2}{\mu^2}\right) \right\} . \end{aligned} \quad (7)$$

where g_A is the pion nucleon coupling constant, f is the pion decay constant, and we have imposed isospin symmetry by setting $m_u = m_d$ and neglecting the electromagnetic interaction. $C_{0,0}$ is a NLO operator with the same structure as C_0 . D_2 is a non-derivative

four-nucleon coupling with one insertion of m_q (or m_π^2), and C_2 is a two-derivative four-nucleon operator that is independent of m_q . μ is the renormalization scale, and we have used dimensional regularization and the power-divergence subtraction procedure (PDS) [59] to renormalize the theory. These amplitudes are manifestly renormalization scale independent order-by-order in the EFT expansion.

Expanding the right hand side of Eq.(4) in powers of p , we have the matching for the spin singlet and triplet scattering lengths

$$\begin{aligned}
\frac{1}{a^{(1S_0)}} &= \gamma^{(1S_0)} - \frac{M}{4\pi}(\gamma^{(1S_0)} - \mu)^2 \left(D_2^{(1S_0)} m_\pi^2 + C_{0,0}^{(1S_0)} \right) \\
&\quad + \frac{g_A^2 M}{8\pi f^2} \left[m_\pi^2 \log \left(\frac{\mu}{m_\pi} \right) + (\gamma^{(1S_0)} - m_\pi)^2 - (\gamma^{(1S_0)} - \mu)^2 \right] \\
\frac{1}{a^{(3S_1)}} &= \gamma^{(3S_1)} - \frac{M}{4\pi}(\gamma^{(3S_1)} - \mu)^2 \left[D_2^{(3S_1)} (m_\pi^2 - \epsilon \lambda^2) + C_{0,0}^{(3S_1)} \right] \\
&\quad + \frac{g_A^2 M}{8\pi f^2} \left[m_\pi^2 \log \left(\frac{\mu}{m_\pi} \right) + (\gamma^{(3S_1)} - m_\pi)^2 - (\gamma^{(3S_1)} - \mu)^2 \right] \\
&\quad - \epsilon \frac{g_A^2 M}{8\pi f^2} \left[\lambda^2 \log \left(\frac{\mu}{\lambda} \right) + (\gamma^{(3S_1)} - \lambda)^2 - (\gamma^{(3S_1)} - \mu)^2 \right], \tag{8}
\end{aligned}$$

where $\gamma^{(i)} = \mu + 4\pi/MC_0^{(i)}$ is the LO inverse scattering length. We perform the expansion around the physical pion mass $m_\pi^{phys} \simeq 138$ MeV, so $\gamma^{(i)}$ takes the physical value $1/a_{phys}^{(i)}$. To fix $D_2^{(i)}$ and $C_{0,0}^{(i)}$, we just need $1/a^{(i)}$ computed at another m_π other than m_π^{phys} .

The matching for effective ranges gives

$$\begin{aligned}
r_0^{(1S_0)} &= \frac{MC_2^{(1S_0)}(\mu - \gamma^{(1S_0)})^2}{2\pi} + \frac{g_A^2 M}{12\pi f^2} \left[6 \left(\frac{\gamma^{(1S_0)}}{m_\pi} \right)^2 - 8 \frac{\gamma^{(1S_0)}}{m_\pi} + 3 \right] \\
r_0^{(3S_1)} &= \frac{MC_2^{(3S_1)}(\mu - \gamma^{(3S_1)})^2}{2\pi} \\
&\quad + \frac{g_A^2 M}{12\pi f^2} \left[6 \left(\frac{\gamma^{(3S_1)}}{m_\pi} \right)^2 - 8 \frac{\gamma^{(3S_1)}}{m_\pi} + 3 - \epsilon \left[6 \left(\frac{\gamma^{(3S_1)}}{\lambda} \right)^2 - 8 \frac{\gamma^{(3S_1)}}{\lambda} + 3 \right] \right]. \tag{9}
\end{aligned}$$

Unlike the scattering lengths, no lattice data is needed to study the quark mass dependence of the effective ranges since $C_2^{(i)}$ can be fixed by $r_0^{(i)}$ at m_π^{phys} .

Note that the ϵ terms in $1/a^{(3S_1)}$ and $r_0^{(3S_1)}$ are m_π independent, so they can be absorbed into counterterms $C_{0,0}^{(3S_1)}$ and $C_2^{(3S_1)}$. Therefore, the KSW and BKV schemes give the same m_π dependence to ERP's at NLO.

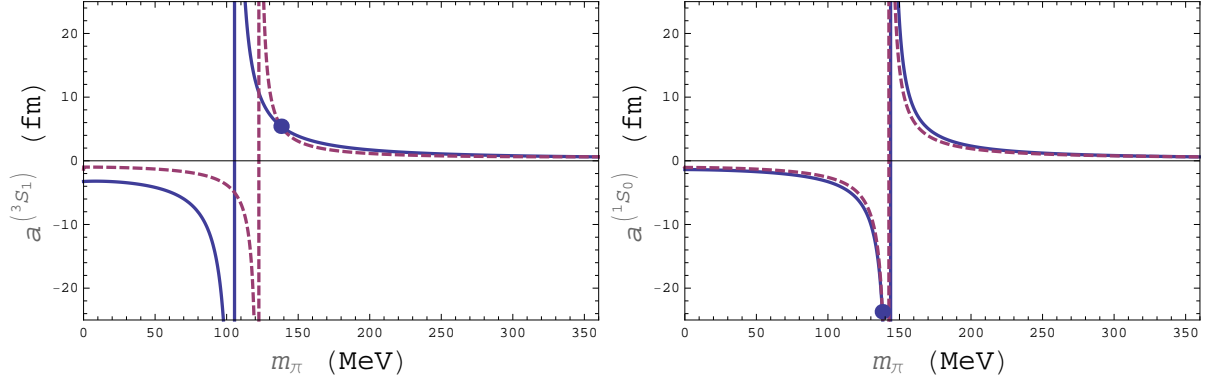


FIG. 1: Scattering lengths of the 3S_1 and 1S_0 states vs. m_π using the NLO BKV result of Eq.(8), the physical scattering length, and the scattering length computed at $m_\pi = 353$ MeV with lattice QCD. The dashed(solid) lines are with(without) the higher order m_π dependence in M , f and g_A included. The dot is the physical point.

In summary, Eqs.(8,9) can be parametrized as

$$\begin{aligned} \frac{1}{a^{(i)}} &= \bar{\gamma}^{(i)} - d_2^{(i)} m_\pi^2 + \frac{g_A^2 M}{8\pi f^2} \left[m_\pi^2 \log\left(\frac{\mu}{m_\pi}\right) + (\gamma^{(i)} - m_\pi)^2 \right] \\ r_0^{(i)} &= c_2^{(i)} + \frac{g_A^2 M}{12\pi f^2} \left[6 \left(\frac{\gamma^{(i)}}{m_\pi} \right)^2 - 8 \frac{\gamma^{(i)}}{m_\pi} \right]. \end{aligned} \quad (10)$$

The physical $a^{(i)}$ and $r_0^{(i)}$ ($a_{phys}^{(^3S_1)} = 5.423 \pm 0.005$ fm, $r_{0,phys}^{(^3S_1)} = 1.764 \pm 0.002$ fm, $a_{phys}^{(^1S_0)} = -23.714 \pm 0.003$ fm, $r_{0,phys}^{(^1S_0)} = 2.73 \pm 0.03$ fm) fix $c_2^{(i)}$ and a combination of $\bar{\gamma}^{(i)}$ and $d_2^{(i)}$. We only need a LQCD calculation of $a^{(i)}$ at different m_π to get the leading m_π dependence for $a^{(i)}$ and $r_0^{(i)}$.

Currently, the smallest m_π that $a^{(i)}$ is computed on the lattice is 353.7 ± 2.1 MeV [23]. The calculation yields $a^{(^3S_1)} = 0.63 \pm 0.74$ fm, $a^{(^1S_0)} = 0.63 \pm 0.50$ fm. The central values yield the solid curves in Fig. 1 and 2. We can study the size of higher order corrections by including the m_π dependence of M , f and g_A (these are next-to-next-to-leading-order corrections) which is extracted from lattice data [65, 66] to Eqs.(8,9). This yields the dashed curves in Fig. 1 and 2. The $a^{(^3S_1)} \rightarrow \infty$ position can shift by $\sim 20\%$ in m_π due to higher order corrections, while the corrections to $a^{(^1S_0)}$ is much smaller. When $m_\pi \gtrsim 100$ MeV, $r_0^{(i)} \simeq 2$ fm and is insensitive to m_π .

The analytic structure of the scattering amplitude, Eq.(1), is that there are two cuts from $p = im_\pi/2$ to $i\infty$ and from $p = -im_\pi/2$ to $-i\infty$. There is a 3S_1 bound state for $m_\pi = 106$ to 142 MeV (with $\lambda = 750$ MeV, but the range remains the same for $\lambda = 500$ to

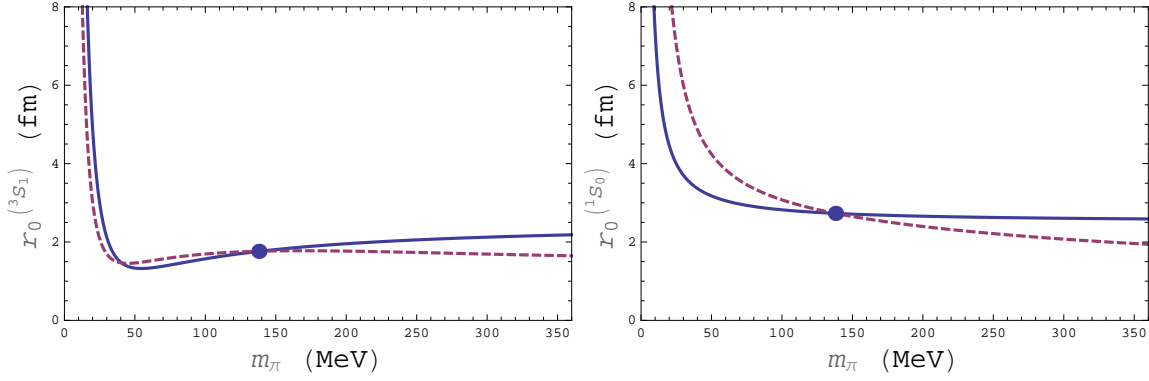


FIG. 2: Effective ranges of the 3S_1 and 1S_0 states vs. m_π using the NLO BKV result of Eq.(9). The notations are the same as in Fig. 1.

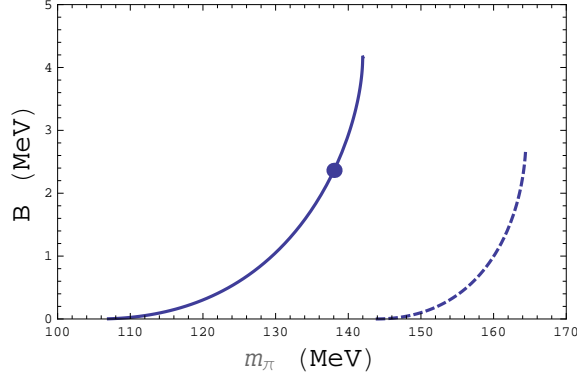


FIG. 3: Bound state (whenever exists) binding energy vs. m_π for 3S_1 (solid line) and 1S_0 (dashed line). The notations are the same as in Fig. 1.

1000 MeV) and a 1S_0 bound state for $m_\pi = 144$ to 165 MeV. The corresponding binding energies are shown in Fig. 3. This result can be understood by examining the scattering amplitude in the effective range expansion. By keeping only the scattering length and effective range in Eq.(2), the amplitude of Eq.(1) has two poles

$$p = \frac{i}{r_0} \left(1 \pm \sqrt{1 - \frac{2r_0}{a}} \right). \quad (11)$$

If $a > 0$, the solution with smaller $|p|$ is $p = \frac{i}{r_0} \left(1 - \sqrt{1 - \frac{2r_0}{a}} \right)$. The bound state exists when $0 < \frac{2r_0}{a} < 1$. On the other hand, if $a < 0$, the solution with smaller $|p|$ is $p = \frac{i}{r_0} \left(1 - \sqrt{1 - \frac{2r_0}{a}} \right)$. Since $\frac{2r_0}{a} < 0$, the pole does not correspond to a bound state. The other pole $p = \frac{i}{r_0} \left(1 + \sqrt{1 - \frac{2r_0}{a}} \right)$ (for both positive and negative a) is of the order of

the ultraviolet cut-off scale $1/r_0$ which is usually hidden in the cut starting at $p = im_\pi/2$. Thus, the bound state range is $0 < 2r_0 < a$, which is close to the ranges seen in Fig. 3. Furthermore, the maximum binding momentum is i/r_0 , or the maximum binding energy is $1/(Mr_0^2) \sim 6$ MeV for $r_0 \sim 2.5$ fm.

With the current lattice input, this theory does not have a two nucleon bound state in the chiral ($m_q \rightarrow 0$, or equivalently $m_\pi \rightarrow 0$) limit. However, different power countings could lead to different conclusions [23]. This might indicate that $m_\pi = 354$ MeV is not within the common “chiral regime”, i.e. within the radius of convergence of the m_π expansion, for these theories. It is important to perform higher order EFT calculations to decide the size of the chiral regime and to answer how small m_π should be for future LQCD calculations to draw a firm conclusion about the deuteron binding energy in the chiral limit.

B. A support for multiverse?

It is curious that m_π^{phys} is so close to the upper bound of m_π where the deuteron is bounded—if m_π^{phys} were 5% bigger, then there would not have been deuteron at all. This makes it much harder for primordial nuclear synthesis to form light nuclei through the usual pathways and might eventually make life impossible. This interesting fine tuning implies that our universe sits near the edge of the parameter space where life could exist. In Ref. [67], it is argued that this is not a fine tuning but a natural case if multiverse exists: In a multiverse, which is an ensemble of many universes including ours, the majority of the universes do not allow life to exist since it requires lots of conditions to be satisfied. Thus, the peak of the m_q distribution in this multiverse will be more likely to sit outside the parameter space where life is possible. In that case, the tail of the distribution goes across this parameter space and then one finds that most of the universes that permits life is near the edge of the parameter space. Thus, if the multiverse exists, without fine tuning, our universe should live near the edge of the parameter space where life is possible (called the catastrophic boundary in [67]). It is interesting to note that our case of the deuteron bound state is consistent with this pattern, similar to the example of the cosmological constant whose value is close to the allowed range obtained by Weinberg through the anthropic principle [68, 69] and several examples worked out in [67]. Although we do not consider this as a sharp test of the multiverse conjecture, because the conjecture cannot be

falsified even if the physical m_q is far away from the edge of the allowed parameter space, it is still interesting to see whether there are cases being consistent with this conjecture.

IV. MATCHING BETWEEN THE THEORY WITH AND WITHOUT PIONS

We are interested in using the theory without pions to describe low energy processes (where $p < m_\pi$ so pions can be integrated out) at non-physical m_π . The matching between the pionful and pionless EFT's at those m_π gives the m_π dependence of the couplings in the pionless theory. Those couplings are the ERP's of NN scattering mentioned above and current operators when coupled to external currents.

We can classify the non-derivative single-nucleon (one-body) current operators by how they transform in the spin-isospin space: the scalar-scalar operator ($N^\dagger N$), scalar-vector operator ($N^\dagger \tau_i N$), vector-scalar operator ($N^\dagger \sigma_i N$) and vector-vector operator ($N^\dagger \sigma_i \tau_j N$), where $\sigma_i(\tau_i)$ acts on the spin(isospin) space and the spacial indexes $i, j = 1, 2, 3$. The non-derivative scalar-scalar and scalar-vector operators originate from matrix elements of the quark level operators $\bar{q}\gamma_0 q$ and $\bar{q}\gamma_0 \tau_i q$. They do not have two-body currents due to vector current conservation. For vector-scalar currents, they could originate from matrix elements of the isoscalar quark axial operator $\bar{q}\gamma_i \gamma_5 q$ or the magnetic part of the vector current $\bar{q}\gamma_i q$, so the corresponding two body-currents exist. For vector-vector currents, the corresponding quark level operator is $\bar{q}\gamma_i \gamma_5 \tau_j q$ and the two body-currents (called Gamow-Teller operators) also exist.

From matching the isoscalar magnetic current between the theory with and without pions, we conclude that the vector-scalar two-body currents do not depend on pion mass at the leading order [70]. The matching of the two-body Gamow-Teller operator [71, 72] yields

$$\begin{aligned}
L_{GT} = l_{GT} &- \frac{\kappa_1 g_A^2 m_\pi^2}{2\gamma^2 f^2} \log\left(\frac{m_\pi}{m_\pi + 2\gamma}\right) \\
&- \frac{\kappa_1 g_A^2}{6a\gamma f^2 m_\pi^2 (m_\pi + 2\gamma)} \left[6a^{(1S_0)} m_\pi^4 + m_\pi^2 \left(9a^{(1S_0)} m_\pi - 4 \right) \gamma - 2m_\pi \gamma^2 \left(a^{(1S_0)} m_\pi - 5 \right) \right. \\
&\quad \left. - 2\gamma^3 \left(5a^{(1S_0)} m_\pi + 6 \right) + 12a^{(1S_0)} \gamma^4 \right], \tag{12}
\end{aligned}$$

where l_{GT} is m_π independent, $\gamma = \left(1 - \sqrt{1 - 2r_0^{(3S_1)}/a^{(3S_1)}} \right) / r_0^{(3S_1)}$ is the deuteron binding momentum and κ_1 is the single nucleon coupling (for the isovector magnetic current, κ_1 is the isovector nucleon magnetic moment; for weak coupling, κ_1 is proportional to

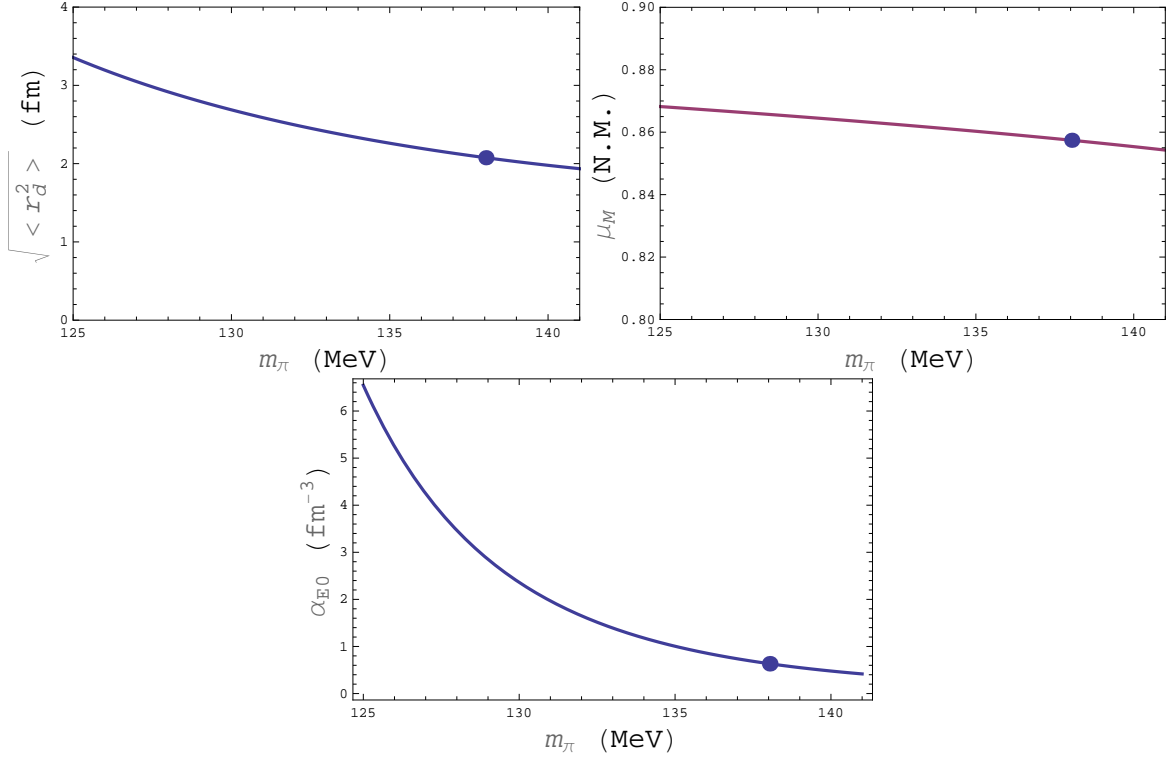


FIG. 4: Deuteron charge radius ($\sqrt{\langle r_d^2 \rangle}$), magnetic moment (μ_M), and electric polarizability (α_{E0}) vs. m_π . The dots are the physical points.

g_A). There is no unknown parameter in the m_π dependent term.

V. THE QUARK MASS DEPENDENCE OF MORE TWO NUCLEON OBSERVABLES

In this section we apply the m_π dependent couplings in the pionless EFT, which has been worked out in the previous sections, to compute several physical observables involving deuterons.

A. Deuteron properties

The deuteron charge radius has the expression [33]

$$\langle r_d^2 \rangle = \langle r_{N,0}^2 \rangle + \frac{1}{8\gamma^2(1 - \gamma\rho_d)} \quad (13)$$

where the isoscalar charge radius of the nucleon $\sqrt{\langle r_{N,0}^2 \rangle} = 0.79 \pm 0.01$ fm and $\rho_d = r_0^{(3S_1)}$. As expected, the deuteron charge radius $\sqrt{\langle r_d^2 \rangle}$ is set by the inverse binding momentum $1/\gamma$ when the nucleon charge radius is negligible. The m_π dependence for $m_\pi = 125 - 141$ MeV (where deuteron is bounded) is shown in Fig. 4.

The deuteron magnetic moment is [33]

$$\mu_M = \frac{e}{2M} (2\kappa_0 + \gamma L_{V-S}), \quad (14)$$

where $\kappa_0 = 0.44$ is the nucleon isoscalar magnetic moment in units of nuclear magneton (N.M.) and the vector-scalar two-body current L_{V-S} is m_π independent. Neither the one-nucleon nor the two-nucleon contribution is sensitive to m_π . The sum is also shown in Fig. 4.

The deuteron polarizability is computed as [33]

$$\alpha_{E,0} = \frac{\alpha M}{32\gamma^4 (1 - \gamma\rho_d)}, \quad (15)$$

$\alpha = 1/137$ is the fine structure constant. It has a strong m_π dependence as is shown in Fig. 4.

B. Reaction: $np \leftrightarrow d\gamma$

The process $np \leftrightarrow d\gamma$ is relevant for BBN. Its cross section is proportional to the wave function overlap between the initial and final states. Since the deuteron size $1/\gamma$ is very sensitive to m_π near m_π^{phys} , the cross section also changes dramatically in this region.

The total cross section for $np \rightarrow d\gamma$ is [73, 74]

$$\sigma(np \rightarrow d\gamma) = \frac{4\pi\alpha(\gamma^2 + p^2)^3}{\gamma^3 M^4 p} \left[|\tilde{X}_{M1}|^2 + |\tilde{X}_{E1}|^2 \right],$$

where p is the magnitude of the momentum of each nucleon in the center-of-mass frame. The electric dipole ($E1$) transition yields

$$|\tilde{X}_{E1}|^2 = \frac{p^2 M^2 \gamma^4}{(\gamma^2 + p^2)^4} [1 + \gamma\rho_d + (\gamma\rho_d)^2 + \dots] \quad (16)$$

The Magnetic dipole ($M1$) transition yields

$$|\tilde{X}_{M1}|^2 = \frac{\kappa_1^2 \gamma^4 \left(\frac{1}{a^{(1S_0)}} - \gamma \right)^2}{\left(\frac{1}{a^{(1S_0)^2}} + p^2 \right) (\gamma^2 + p^2)^2} \left[1 + \gamma\rho_d - r_0^{(1S_0)} \frac{\left(\frac{\gamma}{a^{(1S_0)}} + p^2 \right) p^2}{\left(\frac{1}{a^{(1S_0)^2}} + p^2 \right) \left(\frac{1}{a^{(1S_0)}} - \gamma \right)} - \frac{L_{GT}}{\kappa_1} \frac{M}{2\pi} \frac{\gamma^2 + p^2}{\frac{1}{a^{(1S_0)}} - \gamma} \right].$$

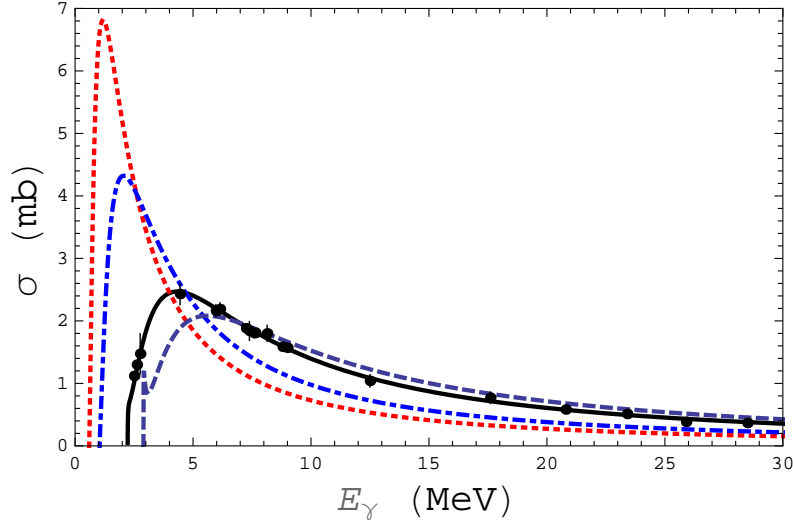


FIG. 5: The cross section for $\gamma d \rightarrow np$ as a function of the incident photon energy in MeV. The solid curve is for the physical m_π , while the dotted, dotdashed, and dashed curves are for $m_\pi = 125, 130$, and 141 MeV, respectively.

The Gamow-Teller two-body current: $L_{GT} = -4.513 \text{ fm}^2$ at $m_\pi = m_\pi^{phys}$, is fitted from the measured cross section $\sigma^{\text{expt}} = 334.2 \pm 0.5 \text{ mb}$ [73] using incident neutrons of speed $|v| = 2200 \text{ m/s}$. The m_π dependence of L_{GT} is shown in Eq.(2). The isovector nucleon magnetic moment $\kappa_1 = 2.35 - g_A^2 M(m_\pi - m_\pi^{phys}) / (2\pi f^2)$, where we have applied the m_π dependence calculated from ChPT [21]. The cross section of the reverse process with deuteron being at rest is

$$\sigma(\gamma d \rightarrow np) = \frac{2M(E_\gamma - B)}{3E_\gamma^2} \sigma(np \rightarrow d\gamma), \quad (17)$$

where E_γ is the incident photon energy. This deuteron photo-disintegration cross section for $m_\pi = 125 - 141 \text{ MeV}$ is shown in Fig. 5.

VI. CONCLUSION

We have studied the implications of lattice QCD determinations of the S-wave nucleon-nucleon scattering lengths at unphysical light quark masses. It is found that with the help of nuclear effective field theory, not only the quark mass dependence of the effective range parameters, but also the leading quark mass dependence of all the low energy (with $p \ll m_\pi$) deuteron matrix elements can be obtained. The quark mass dependence of

deuteron charge radius, magnetic moment, polarizability and the deuteron photodisintegration cross section are shown based on the NPLQCD lattice calculation of the scattering lengths at 354 MeV pion mass and the NEFT power counting scheme of Beane, Kaplan and Vuorinen. Further improvement can be obtained by performing the lattice calculation at smaller quark masses. But at the same time, it is important to perform higher order EFT calculations to decide the radius of convergence in m_π in order to answer how small m_π should be for future LQCD calculations to provide reliable m_π dependence for two nucleon observables all the way to the chiral limit.

Our result can be used to constrain the time variation of isoscalar combination of u and d quark mass m_q , to help the anthropic principle study to find the m_q range which allows the existence of life, and to provide a weak test of the multiverse conjecture.

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